GO #6: Quadratic Functions

Vertex Form: $f(x) = a(x - h)^2 + k$ Standard Form: $f(x) = ax^2 + bx + c$

Converting Quadratic Equations from vertex form into standard form:

Vertex Form: Square the binomial.

Foil or Box Method

	x	-4
x	x^2	$-4x$
-4	$-4x$	16

$x^2 - 4x - 4x + 16$

$f(x) = -2(x - 4)^2 + 5$

$(x - 4)(x - 4)$ FOIL

$x^2 - 4x - 4x + 16$

$x^2 - 8x + 16$

Distribute the coefficient of the trinomial..

$= -2(x^2 - 8x + 16) + 5$

Combine like terms.

$= -2x^2 + 16x - 32 + 5$

$= -2x^2 + 16x - 27$

Standard Form

Convert from Vertex Form to Standard Form:

$$y = a(x-h)^2 + k \quad \Rightarrow \quad y = ax^2 + bx = c$$

Example 1:

$\mathbf{y = 5(x+2)^2 - 9}$		
$y = 5(x+2)(x+2) - 9$	⇐	Rewrite $(x+2)^2$
$y = 5(x^2 + 4x + 4) - 9$	⇐	Simplify $(x+2)(x+2)$
$y = 5x^2 + 20x + 20 - 9$	⇐	Distribute the 5
$\mathbf{y = 5x^2 + 20x + 11}$	⇐	Combine Like Terms

$(x+2)(x+2)$ foil or box

Example 2:

$\mathbf{y = -3(x-4)^2 + 7}$		
$y = -3(x-4)(x-4) + 7$	⇐	Rewrite $(x-4)^2$
$y = -3(x^2 - 8x + 16) + 7$	⇐	Simplify $(x-4)(x-4)$
$y = -3x^2 + 24x - 48 + 7$	⇐	Distribute the -3
$\mathbf{y = -3x^2 + 24x - 41}$	⇐	Combine Like Terms

$(x-4)(x-4)$ foil or box

Practice: *Convert the following quadratics from vertex to standard form.*

1. $y = (x-2)^2 + 6$
2. $y = 3(x-3)^2 - 12$
3. $y = -2(x+1)^2 + 3$

Advanced Math
Worksheet—Vertex Form to Standard Form

Name ____________________________
Date ______________ Hour ________

We have been working with quadratic equations in Vertex Form, $y = a(x-h)^2 + k$. However, it is more common for quadratic equations to be given to us in Standard Form, $y = ax^2 + bx + c$. Today's assignment is for you to practice using FOIL to change equations from Vertex Form into Standard Form. Use the example below to guide your work.

Example:

$y = -2(x+3)^2 - 5$	Given.
$y = -2(x^2 + 6x + 9) - 5$	Multiply the quantity squared. (FOIL)
$y = -2x^2 - 12x - 18 - 5$	Distribute the a.
$y = -2x^2 - 12x - 23$	Combine like terms.

(x+3)(x+3)

$\cdot$	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

or

$x^2 + 3x + 3x + 9$

Problems:

1. $y = 6(x-4)^2 - 1$	2. $y = \frac{1}{2}(x+4)^2 + 6$	3. $y = -5(x-1)^2 + 4$
4. $y = -\frac{1}{3}(x+6)^2 - 1$	5. $y = 4(x+2)^2 - 8$	6. $y = \frac{-2}{3}(x-9)^2 - 2$
7. $y = (x-2)^2 + 7$	8. $y = (x+\frac{1}{2})^2 - 2$	9. $y = 18(x-\frac{1}{3})^2 + 5$
10. $y = -2\left(x+\frac{1}{2}\right)^2$	11. $y = 13(x-2)^2 + 15$	12. $y = 2(x+8)^2 + 10$

Algebra
9.3 & 9.4 Notes
Solving & Graphing Quadratic Functions

Standards:
A1.8.1 Graph quadratic, cubic, and radical equations.
A1.8.7 Use quadratic equations to solve word problems.

9.3 Vocabulary

1. A **quadratic function** is a function that can be written in the **standard form:**

   $$y = ax^2 + bx + c, \text{ where } a \neq 0$$

2. Every quadratic function has a U-shaped graph called a ____________________.

3. If the leading coefficient a is positive, the parabola ____________________.

4. If the leading coefficient a is negative, the parabola ____________________.

5. The ____________ is the lowest point of a parabola that opens up and the highest point of a parabola that opens down.

6. The line passing through the vertex that divides the parabola into two symmetric parts is called the ____________________________.

7. Solutions of quadratic functions can also be called the __________________, __________________, or __________________.

To find the Vertex and Axis of Symmetry

1. Put the quadratic function in standard form: $y = ax^2 + bx + c$
2. Identify the numeric values of a, b, and c.
3. The vertex has an x-coordinate of $x = \frac{-b}{2a}$. Plug in the values for a and b.
4. Substitute whatever you get for x in step 3 into the quadratic function to find the y-coordinate of the vertex.
5. The axis of symmetry is the vertical line $x = \frac{-b}{2a}$

Homework: 9.3/9.4 Worksheet Day 1

Algebra
9.3 & 9.4 Notes
Solving & Graphing Quadratic Functions

Example 1 – Find the vertex and Axis of Symmetry for these quadratic functions:	
a.) $y = -2x^2 + 4x - 9$ $a =$ ___ $b =$ ___ $c =$ ___ Vertex: __________ Axis of Symmetry: __________	b.) $y = x^2 - 10$ $a =$ ___ $b =$ ___ $c =$ ___ Vertex: __________ Axis of Symmetry: __________
c.) $y = x^2 + 4x - 1$ $a =$ ___ $b =$ ___ $c =$ ___ Vertex: __________ Axis of Symmetry: __________	d.) $y = -2x^2 + 8x - 8$ $a =$ ___ $b =$ ___ $c =$ ___ Vertex: __________ Axis of Symmetry: __________

Steps for Graphing a Quadratic Function

1. Follow the above steps to find the vertex and axis of symmetry.
2. Plot the vertex and the axis of symmetry on a coordinate plane.
3. Make a table of values, using x-values to the left and right of the vertex.
4. Plot the points and connect them with a smooth curve to form a parabola.

Homework: 9.3/9.4 Worksheet Day 1

Converting Quadratic Equations between Standard and Vertex Form

Standard Form: $y = ax^2 + bx + c$ Vertex Form: $y = a(x - h)^2 + k$

Convert from Standard Form to Vertex Form:

$y = ax^2 + bx = c \implies y = a(x-h)^2 + k$

know a, b, c want a, h, k

★ $X = \frac{-b}{2a}$

a = a

$x = \frac{-b}{2a}$ = h

Solve for y = k

Substitute the values and rewrite.

Example 1:

$y = 8x^2 - 16x + 27$

We know *a*, *b*, *c* and want *a*, *h*, *k*

$a = 8$ b= -16 C = 27 ⟸ ***a*** is the coefficient of the x^2 term

$h = x = \frac{-b}{2a} = \frac{-(-16)}{2(8)} = \frac{16}{16} = 1$ ⟸ use the formula to find the value of ***h***

$k = y = 8(1)^2 - 16(1) + 27 = 8 - 16 + 27 = 19$ ⟸ substitute the value found for ***h*** into the original equation and solve for ***k***

$y = 8(x - 1)^2 + 19$

Example 2:

$y = 5x^2 - 40x + 67$

We know *a*, *b*, *c* and want *a*, *h*, *k*

$a = 5$ ⟸ ***a*** is the coefficient of the x^2 term

$h = x = \frac{-b}{2a} = \frac{-(-40)}{2(5)} = \frac{40}{10} = 4$ ⟸ use the formula to find the value of ***h***

$k = y = 5(4)^2 - 40(4) + 67 = 80 - 160 + 67 = -13$ ⟸ substitute the value found for ***h*** into the original equation and solve for ***k***

$y = 5(x - 4)^2 - 13$

Practice: *Convert the following quadratics from standard to vertex form.*

1. $y = 5x^2 - 10x + 37$
2. $y = 7x^2 + 28x + 19$
3. $y = -2x^2 - 24x - 75$

Unit 1 Lesson 1 Name________________________________

1. Convert from standard form to vertex form. 2. Identify vertex and axis of symmetry. (Work on notebook paper & answer in box)

1. $4x^2 + 40x + 3 = 0$
2. $-x^2 + 6x + 4 = 0$
3. $x^2 + 4x + 2 = 0$
4. $-2x^2 + 4x + 11 = 0$
5. $3x^2 - 6x + 8 = 0$
6. $-4x^2 - 24x + 9 = 0$
7. $-x^2 - 10x + 4 = 0$
8. $2x^2 + 20x + 1 = 0$
9. $-x^2 - 2x + 11 = 0$
10. $-3x^2 + 6x - 4 = 0$
11. $-2x^2 + 4x - 5 = 0$
12. $2x^2 - 16x - 3 = 0$
13. $x^2 - 4x + 2 = 0$
14. $3x^2 + 18x + 5 = 0$
15. $4x^2 - 40x - 1 = 0$

Identify 1. axis of symmetry 2. Vertex (Work on notebook paper & answer in box)

1. $5x^2 - 2x - 6 = 0$
2. $-8x^2 + 9x + 4 = 0$
3. $5x^2 + 10x + 2 = 0$
4. $6x^2 + 11x - 12 = 0$
5. $-x^2 + 9x - 6 = 0$
6. $-12x^2 - x + 10 = 0$
7. $7x^2 + 9x - 6 = 0$
8. $8x^2 - 11x - 2 = 0$
9. $-x^2 - 2x + 5 = 0$
10. $-8x^2 + 9x - 1 = 0$
11. $-6x^2 + 11x - 3 = 0$
12. $-4x^2 + x + 2 = 0$
13. $-11x^2 - 9x + 5 = 0$
14. $-2x^2 + 5x + 4 = 0$
15. $x^2 + 6x - 5 = 0$

Standard form: $y = ax^2 + bx + c$ Vertex form: $y = a(x-h)^2 + k$

More Vertex Form Worksheet

Using the same processes we developed in "Vertex Form Begun," rewrite each of these quadratic equations.

Expand each quadratic and write in Standard Form. Identify the Vertex for each: (?, ?)

Vertex Form	Standard Form	Vertex is at . . .
1. $y=(x+3)^2-10$ a= h= k=		
2. $y=(x-5)^2+4$ a= h= k=		
3. $y=(x+\frac{2}{3})^2+\frac{2}{9}$ a= h= k=		
4. $y=2(x+1)^2-7$ a= h= k=		

Now, take each of these and rewrite in Vertex Form. Then identify the vertex: (?, ?) $x = \frac{-b}{2a}$

Standard Form	Vertex Form	Vertex is at . . .
5. $y=x^2+8x-1$ a= b= c=		
6. $y=x^2-6x+17$ a= b= c=		
7. $y=x^2-5x-11$ a= b= c=		
8. $y=x^2+10x$ a= b= c=		
9. $y=x^2+bx+c$ a= b= c=		

Quadratic Transformation Worksheet Name____________________________

1. Write the vertex form of a quadratic equation.

2. What does changing the "a" variable do to the graph of a quadratic?

3. Being specific, name 3 ways that a parabola changes with different types of "a" values.

4. What does changing the "h" variable do to the graph of a quadratic?

5. If "h" is positive how does the parabola move? If negative?

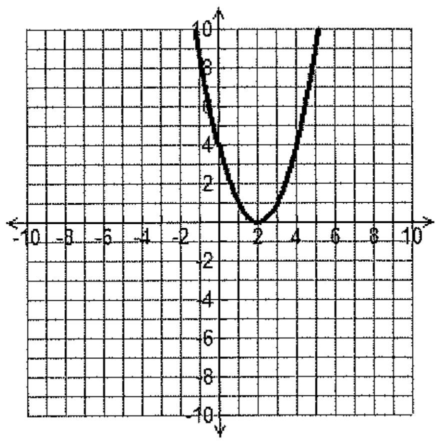
6. What does changing the "k" variable do to the graph of a quadratic?

7. If "k" is positive how does the parabola move? If negative?

8. What conclusion can you make about the variables of h and k together?

Write the quadratic equation, in vertex form for each graph.

1.



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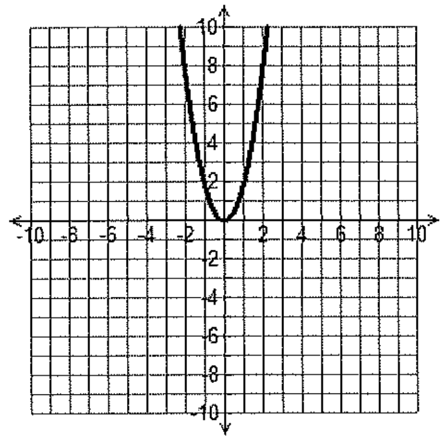

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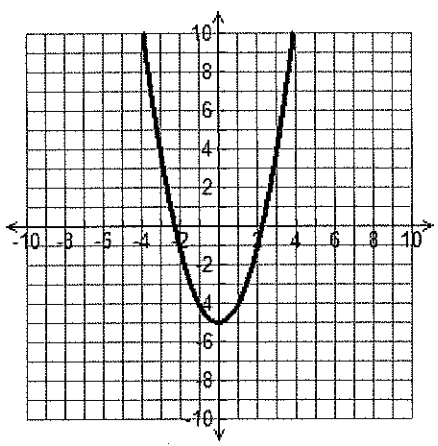

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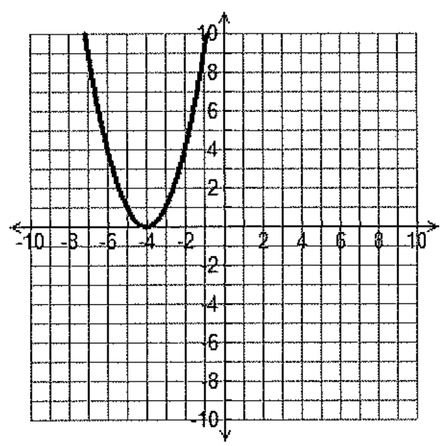

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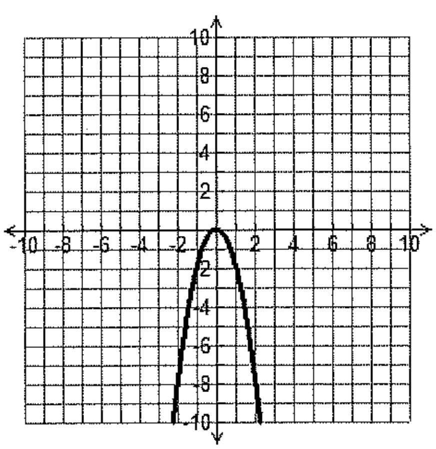

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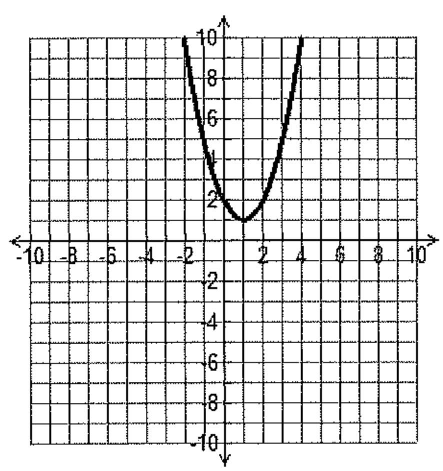

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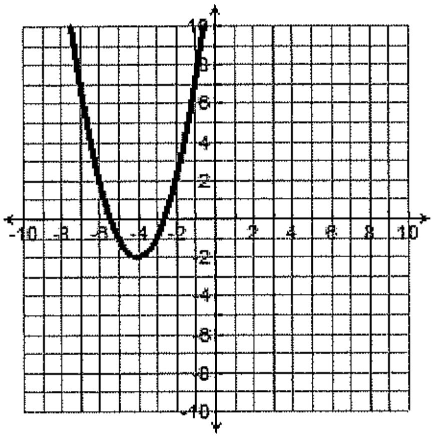

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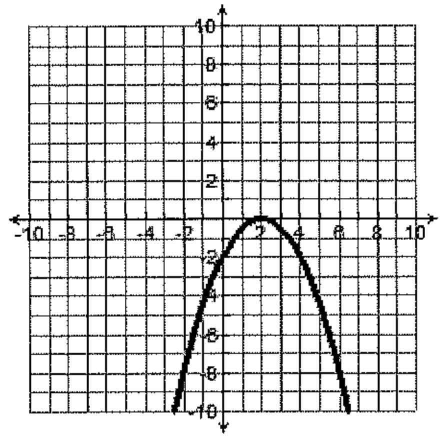

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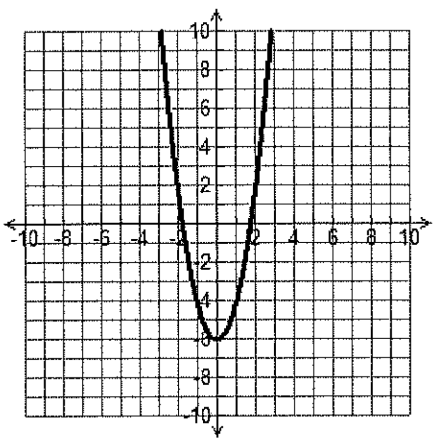

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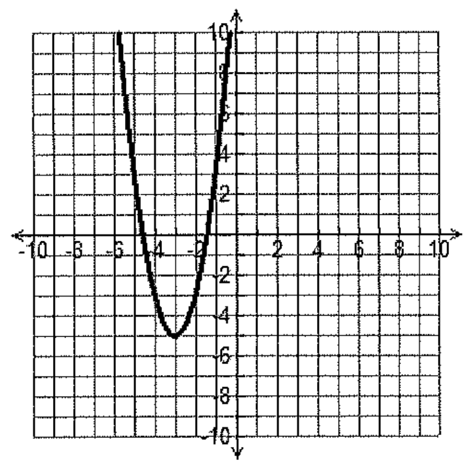
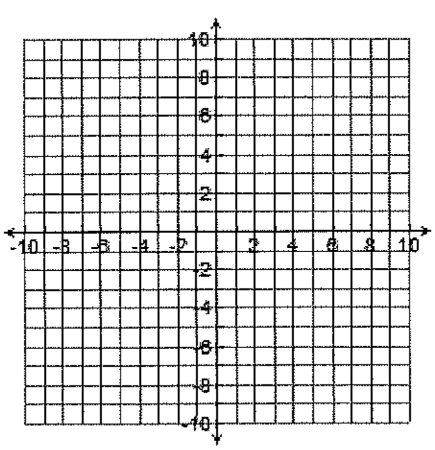
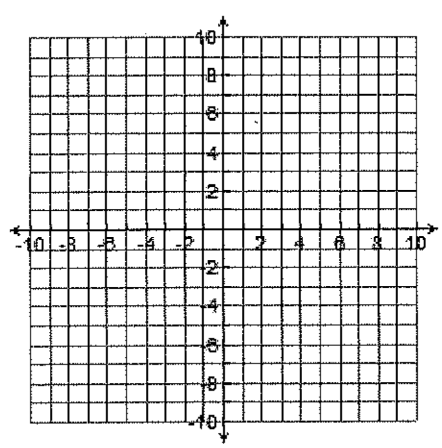


Graph the quadratic equation on the provided grid.

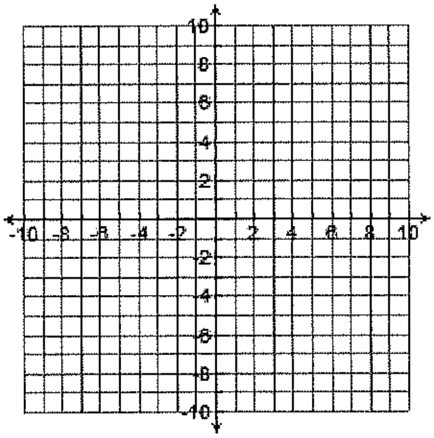
11. $f(x) = (x-0)^2 + 3$



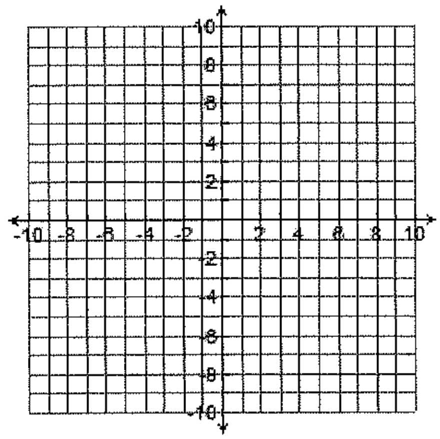
12. $f(x) = (x+4)^2 + 0$



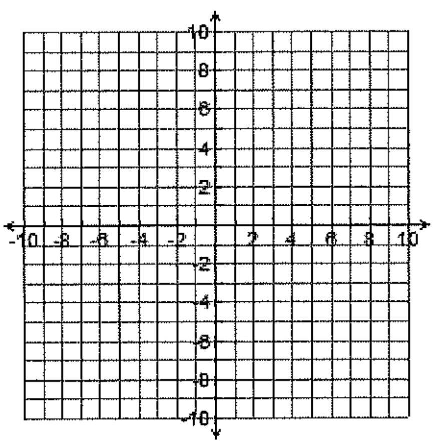
13. $f(x) = -2(x-0)^2 + 0$



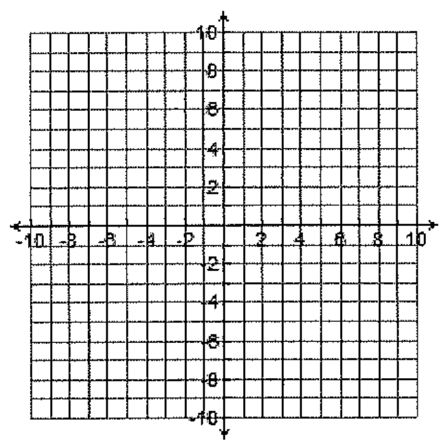
14. $f(x) = (x-3)^2 + 4$



15. $f(x) = 3(x-4)^2 - 6$

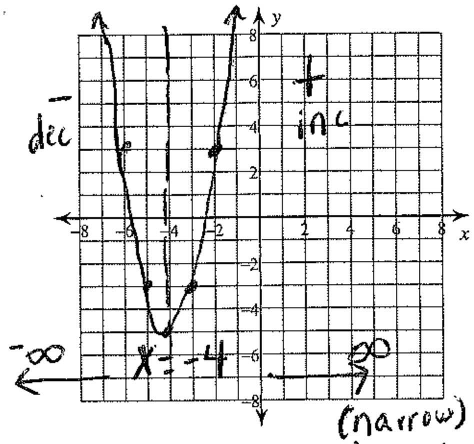


16. $f(x) = \frac{1}{2}(x+2)^2 + 3$



Identify the vertex and axis of symmetry of each. Then sketch the graph.

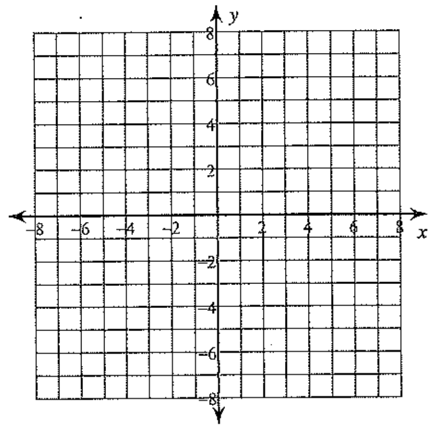
11) $f(x) = 2(x+4)^2 - 5$



X	y
-2	3
-3	-3
-4	-5 vertex
-5	-3
-6	3

y intercept: 27
(0, 27)

12) $f(x) = -\frac{1}{4}(x+4)^2 - 5$



Transformation: Vert. Stretch left 4 down 5

domain: $(-\infty, \infty)$ $\mathbb{R}$ $-\infty < x < \infty$

range: $[-5, \infty)$ $y \geq -5$

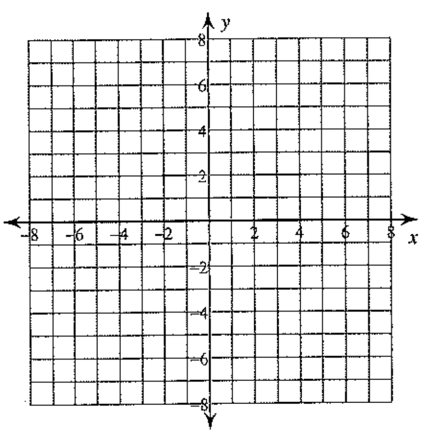
vertex: $(-4, -5)$ max or (min) $(-4, -5)$

a.o.s.: $x = -4$

Inc: $(-4, \infty)$ $x > -4$ $-4 < x < \infty$

dec: $(-\infty, -4)$ $x < -4$ $-\infty < x < -4$

13) $f(x) = -(x-6)^2 + 2$



14) $f(x) = -(x-5)^2 - 1$